2 - 7 General solution
Find a general solution.

3. $y_1' = y_2 + e^{3t}$
$y_2' = y_1 - 3e^{3t}$

```
ClearAll["Global`*"]
```

As in the last section, there will be rearranging, recasting, and substitutions after the solution appears in order to make its form like the text answer.

```
e1 = {y1'[t] == y2[t] + e^(3t), y2'[t] == y1[t] - 3 e^(3t)}
e2 = DSolve[e1, {y1, y2}, t]
```

$\{y1'[t] == e^{3t} + y2[t],\ y2'[t] == -3 e^{3t} + y1[t]\}$

$\{\{y1 \to \text{Function}[\{t\}, \frac{1}{2} e^{-t} (1 + e^{2t})\, C[1] + \frac{1}{2} e^{-t} (-1 + e^{2t})\, C[2]],$
$y2 \to \text{Function}[\{t\}, \frac{1}{4} e^{t} (-1 + e^{2t})^2 - \frac{1}{4} e^{t} (1 + e^{2t})^2 +$
$\frac{1}{2} e^{-t} (-1 + e^{2t})\, C[1] + \frac{1}{2} e^{-t} (1 + e^{2t})\, C[2]]\}\}$

```
e3 = e2[[1, 1, 2, 2]]
```

$\frac{1}{2} e^{-t} (1 + e^{2t})\, C[1] + \frac{1}{2} e^{-t} (-1 + e^{2t})\, C[2]$

```
e4 = Expand[e3]
```

$\frac{1}{2} e^{-t} C[1] + \frac{1}{2} e^{t} C[1] - \frac{1}{2} e^{-t} C[2] + \frac{1}{2} e^{t} C[2]$

```
e5 = Collect[e4, e^-t]
```

$e^{-t} \left(\frac{C[1]}{2} - \frac{C[2]}{2}\right) + e^{t} \left(\frac{C[1]}{2} + \frac{C[2]}{2}\right)$

$e6 = e5\ /.\ \{\left(\frac{C[1]}{2} - \frac{C[2]}{2}\right) \to c1,\ \left(\frac{C[1]}{2} + \frac{C[2]}{2}\right) \to c2\}$

$c1\, e^{-t} + c2\, e^{t}$

```
e7 = e2[[1, 2, 2, 2]]
```

$\frac{1}{4} e^{t} (-1 + e^{2t})^2 - \frac{1}{4} e^{t} (1 + e^{2t})^2 + \frac{1}{2} e^{-t} (-1 + e^{2t})\, C[1] + \frac{1}{2} e^{-t} (1 + e^{2t})\, C[2]$

```
e8 = Expand[e7]
```

$$-e^{3t} - \frac{1}{2} e^{-t} C[1] + \frac{1}{2} e^{t} C[1] + \frac{1}{2} e^{-t} C[2] + \frac{1}{2} e^{t} C[2]$$

```
e9 = Collect[e8, e^t]
```

$$-e^{3t} + e^{-t}\left(-\frac{C[1]}{2} + \frac{C[2]}{2}\right) + e^{t}\left(\frac{C[1]}{2} + \frac{C[2]}{2}\right)$$

$$e10 = e9 /. \left\{\left(-\frac{C[1]}{2} + \frac{C[2]}{2}\right) \to -c1,\ \left(\frac{C[1]}{2} + \frac{C[2]}{2}\right) \to c2\right\}$$

$$-c1\, e^{-t} + c2\, e^{t} - e^{3t}$$

1. Above: The expressions in the green cells match the text answers for y_1 and y_2 respectively. The substitutions of symbolic constants shown in the yellow cells match values of constants between the two function expressions, demonstrating that the constant substitution system is self-consistent and consistent with the text.

5. $y_1' = 4 y_1 + y_2 + 0.6 t$
$y_2' = 2 y_1 + 3 y_2 - 2.5 t$

```
ClearAll["Global`*"]
```

```
e1 = {y1'[t] == 4 y1[t] + y2[t] + 0.6 t, y2'[t] == 2 y1[t] + 3 y2[t] - 2.5 t}
e2 = DSolve[e1, {y1, y2}, t]
```

```
{y1′[t] == 0.6 t + 4 y1[t] + y2[t], y2′[t] == -2.5 t + 2 y1[t] + 3 y2[t]}
```

$$\{\{y1 \to \text{Function}[\{t\},$$
$$-0.333333\,(1.\,e^{2.t} - 1.\,e^{5.t})\,(-2.06667\,e^{-2.t}\,(-0.25 - 0.5\,t) - 0.433333\,e^{-5.t}\,(-0.04 - 0.2\,t)) + 0.333333\,(1.\,e^{2.t} + 2.\,e^{5.t})$$
$$(1.03333\,e^{-2.t}\,(-0.25 - 0.5\,t) - 0.433333\,e^{-5.t}\,(-0.04 - 0.2\,t)) +$$
$$0.333333\,(1.\,e^{2.t} + 2.\,e^{5.t})\,C[1] - 0.333333\,(1.\,e^{2.t} - 1.\,e^{5.t})\,C[2]],$$
$$y2 \to \text{Function}[\{t\},\ 0.666667\,(1.\,e^{2.t} + 0.5\,e^{5.t})$$
$$(-2.06667\,e^{-2.t}\,(-0.25 - 0.5\,t) - 0.433333\,e^{-5.t}\,(-0.04 - 0.2\,t)) -$$
$$0.666667\,(1.\,e^{2.t} - 1.\,e^{5.t})$$
$$(1.03333\,e^{-2.t}\,(-0.25 - 0.5\,t) - 0.433333\,e^{-5.t}\,(-0.04 - 0.2\,t)) -$$
$$0.666667\,(1.\,e^{2.t} - 1.\,e^{5.t})\,C[1] + 0.666667\,(1.\,e^{2.t} + 0.5\,e^{5.t})\,C[2]]\}\}$$

```
e3 = e2[[1, 1, 2, 2]]
```

$$-0.333333\left(1.\,e^{2.\,t}-1.\,e^{5.\,t}\right)\left(-2.06667\,e^{-2.\,t}\,(-0.25-0.5\,t)-0.433333\,e^{-5.\,t}\,(-0.04-0.2\,t)\right)+0.333333\left(1.\,e^{2.\,t}+2.\,e^{5.\,t}\right)\left(1.03333\,e^{-2.\,t}\,(-0.25-0.5\,t)-0.433333\,e^{-5.\,t}\,(-0.04-0.2\,t)\right)+0.333333\left(1.\,e^{2.\,t}+2.\,e^{5.\,t}\right)C[1]-0.333333\left(1.\,e^{2.\,t}-1.\,e^{5.\,t}\right)C[2]$$

```
e4 = Simplify[e3]
```

$$e^{-3.\,t}\left(-8.67362\times10^{-19}+e^{3.\,t}\,(-0.241-0.43\,t)+e^{6.\,t}\left(-2.77556\times10^{-17}-5.55112\times10^{-17}\,t\right)+e^{5.\,t}\,(0.333333\,C[1]-0.333333\,C[2])+e^{8.\,t}\,(0.666667\,C[1]+0.333333\,C[2])\right)$$

```
e5 = Expand[e4]
```

$$-0.241-8.67362\times10^{-19}\,e^{-3.\,t}-2.77556\times10^{-17}\,e^{3.\,t}-0.43\,t-5.55112\times10^{-17}\,e^{3.\,t}\,t+0.333333\,e^{2.\,t}\,C[1]+0.666667\,e^{5.\,t}\,C[1]-0.333333\,e^{2.\,t}\,C[2]+0.333333\,e^{5.\,t}\,C[2]$$

```
e6 = Chop[e5, 10^-16]
```

$$-0.241-0.43\,t+0.333333\,e^{2.\,t}\,C[1]+0.666667\,e^{5.\,t}\,C[1]-0.333333\,e^{2.\,t}\,C[2]+0.333333\,e^{5.\,t}\,C[2]$$

```
e7 = Collect[e6, {ⅇ^(2. t), ⅇ^(5. t)}]
```

$$-0.241-0.43\,t+e^{2.\,t}\,(0.333333\,C[1]-0.333333\,C[2])+e^{5.\,t}\,(0.666667\,C[1]+0.333333\,C[2])$$

```
e8 = e7 /. {(0.3333333333333334` C[1] - 0.3333333333333333` C[2]) → c2,
    (0.6666666666666666` C[1] + 0.3333333333333333` C[2]) → c1}
```

$$-0.241+c2\,e^{2.\,t}+c1\,e^{5.\,t}-0.43\,t$$

```
e9 = e2[[1, 2, 2, 2]]
```

$$0.666667\left(1.\,e^{2.\,t}+0.5\,e^{5.\,t}\right)\left(-2.06667\,e^{-2.\,t}\,(-0.25-0.5\,t)-0.433333\,e^{-5.\,t}\,(-0.04-0.2\,t)\right)-0.666667\left(1.\,e^{2.\,t}-1.\,e^{5.\,t}\right)\left(1.03333\,e^{-2.\,t}\,(-0.25-0.5\,t)-0.433333\,e^{-5.\,t}\,(-0.04-0.2\,t)\right)-0.666667\left(1.\,e^{2.\,t}-1.\,e^{5.\,t}\right)C[1]+0.666667\left(1.\,e^{2.\,t}+0.5\,e^{5.\,t}\right)C[2]$$

```
e10 = Simplify[e9]
```

$$0.534+1.73472\times10^{-18}\,e^{-3.\,t}+1.12\,t+e^{5.\,t}\,(0.666667\,C[1]+0.333333\,C[2])+e^{2.\,t}\,(-0.666667\,C[1]+0.666667\,C[2])$$

```
e11 = Chop[e10, 10^-16]
```

$0.534 + 1.12\,t + e^{5.\,t}\,(0.666667\,C[1] + 0.333333\,C[2]) + e^{2.\,t}\,(-0.666667\,C[1] + 0.666667\,C[2])$

```
e12 = e11 /. {(0.6666666666666669` C[1] + 0.3333333333333334` C[2]) → c1,
    (-0.6666666666666669` C[1] + 0.6666666666666667` C[2]) → -2 c2}
```

$0.534 - 2\,c2\,e^{2.\,t} + c1\,e^{5.\,t} + 1.12\,t$

1. Above: The expressions in the green cells match the text answers for y_1 and y_2 respectively. The substitutions of symbolic constants shown in the yellow cells match values of constants between the two function expressions, demonstrating that the constant substitution system is self-consistent and consistent with the text.

7. $y_1' = -3\,y_1 - 4\,y_2 + 11\,t + 15$
$y_2' = 5\,y_1 + 6\,y_2 + 3\,e^{-t} - 15\,t - 20$

```
ClearAll["Global`*"]
```

```
e1 = {y1'[t] == -3 y1[t] - 4 y2[t] + 11 t + 15,
   y2'[t] == 5 y1[t] + 6 y2[t] + 3 e^-t - 15 t - 20}
e2 = DSolve[e1, {y1, y2}, t]
```

$\{y1'[t] == 15 + 11\,t - 3\,y1[t] - 4\,y2[t],\ y2'[t] == -20 + 3\,e^{-t} - 15\,t + 5\,y1[t] + 6\,y2[t]\}$

$\{\{y1 \to \text{Function}[\{t\}, -e^{t}\,(-5 + 4\,e^{t})\,(4\,e^{-3t} + e^{-2t}\,(-20 - 8\,t) + e^{-t}\,(10 + 5\,t)) - 4\,e^{t}\,(-1 + e^{t})\,\left(-5\,e^{-3t} + e^{-t}\,(-10 - 5\,t) + e^{-2t}\left(\frac{47}{2} + 10\,t\right)\right) - e^{t}\,(-5 + 4\,e^{t})\,C[1] - 4\,e^{t}\,(-1 + e^{t})\,C[2]],$
$y2 \to \text{Function}[\{t\}, 5\,e^{t}\,(-1 + e^{t})\,(4\,e^{-3t} + e^{-2t}\,(-20 - 8\,t) + e^{-t}\,(10 + 5\,t)) + e^{t}\,(-4 + 5\,e^{t})\,\left(-5\,e^{-3t} + e^{-t}\,(-10 - 5\,t) + e^{-2t}\left(\frac{47}{2} + 10\,t\right)\right) + 5\,e^{t}\,(-1 + e^{t})\,C[1] + e^{t}\,(-4 + 5\,e^{t})\,C[2]]\}\}$

```
e3 = e2[[1, 1, 2, 2]]
```

$-e^{t}\,(-5 + 4\,e^{t})\,(4\,e^{-3t} + e^{-2t}\,(-20 - 8\,t) + e^{-t}\,(10 + 5\,t)) - 4\,e^{t}\,(-1 + e^{t})\,\left(-5\,e^{-3t} + e^{-t}\,(-10 - 5\,t) + e^{-2t}\left(\frac{47}{2} + 10\,t\right)\right) - e^{t}\,(-5 + 4\,e^{t})\,C[1] - 4\,e^{t}\,(-1 + e^{t})\,C[2]$

```
e4 = Simplify[e3]
```

$e^{-t}\,(-2 - e^{t}\,(4 + 3\,t) - 4\,e^{3t}\,(C[1] + C[2]) + e^{2t}\,(5\,C[1] + 4\,C[2]))$

```
e5 = Expand[e4]
```

$-4 - 2 e^{-t} - 3 t + 5 e^{t} C[1] - 4 e^{2t} C[1] + 4 e^{t} C[2] - 4 e^{2t} C[2]$

```
e6 = Collect[e5, {e^(2t), e^t}]
```

$-4 - 2 e^{-t} - 3 t + e^{2t} (-4 C[1] - 4 C[2]) + e^{t} (5 C[1] + 4 C[2])$

```
e7 = e6 /. {(-4 C[1] - 4 C[2]) → 4 c2, (5 C[1] + 4 C[2]) → c1}
```

$-4 - 2 e^{-t} + c1\, e^{t} + 4\, c2\, e^{2t} - 3 t$

```
e8 = e2[[1, 2, 2, 2]]
```

$5 e^{t} (-1 + e^{t}) (4 e^{-3t} + e^{-2t} (-20 - 8 t) + e^{-t} (10 + 5 t)) + e^{t} (-4 + 5 e^{t}) \left(-5 e^{-3t} + e^{-t} (-10 - 5 t) + e^{-2t} \left(\frac{47}{2} + 10 t\right)\right) + 5 e^{t} (-1 + e^{t}) C[1] + e^{t} (-4 + 5 e^{t}) C[2]$

```
e9 = Simplify[e8]
```

$\frac{15}{2} + e^{-t} + 5 t + 5 e^{2t} (C[1] + C[2]) - e^{t} (5 C[1] + 4 C[2])$

```
e10 = e9 /. {(C[1] + C[2]) → -c2, (5 C[1] + 4 C[2]) → c1}
```

$\frac{15}{2} + e^{-t} - c1\, e^{t} - 5\, c2\, e^{2t} + 5 t$

1. Above: The expressions in the green cells match the text answers for y_1 and y_2 respectively. The substitutions of symbolic constants shown in the yellow cells match values of constants between the two function expressions, demonstrating that the constant substitution system is self-consistent and consistent with the text.

10 - 15 Initial value problem
Solve, showing details.

11. $y_1' = y_2 + 6 e^{2t}$
$y_2' = y_1 - e^{2t}$
$y_1[0] = 1,\ y_2[0] = 0$

```
ClearAll["Global`*"]
```

```
e1 = {y1'[t] == y2[t] + 6 ⅇ^(2 t), y2'[t] == y1[t] - ⅇ^(2 t), y1[0] == 1, y2[0] == 0}
e2 = DSolve[e1, {y1, y2}, t]
```

$\{y1'[t] == 6\,e^{2t} + y2[t],\ y2'[t] == -e^{2t} + y1[t],\ y1[0] == 1,\ y2[0] == 0\}$

$\{\{y1 \to \text{Function}[\{t\},\ \frac{1}{3} e^{-t} (-2 - 6\,e^{2t} + 11\,e^{3t})],$
$y2 \to \text{Function}[\{t\},\ \frac{2}{3} e^{-t} (-1 + e^{t})^2 (1 + 2\,e^{t})]\}\}$

```
e3 = e2[[1, 1, 2, 2]]
```

$\frac{1}{3} e^{-t} (-2 - 6\,e^{2t} + 11\,e^{3t})$

```
e4 = Expand[e3]
```

$-\frac{2\,e^{-t}}{3} - 2\,e^{t} + \frac{11\,e^{2t}}{3}$

```
e5 = e4 /. (-2 ⅇ^-t/3 - 2 ⅇ^t) → ExpToTrig[-2 ⅇ^-t/3 - 2 ⅇ^t]
```

$\frac{11\,e^{2t}}{3} - \frac{8\,\text{Cosh}[t]}{3} - \frac{4\,\text{Sinh}[t]}{3}$

```
e6 = e2[[1, 2, 2, 2]]
```

$\frac{2}{3} e^{-t} (-1 + e^{t})^2 (1 + 2\,e^{t})$

```
e7 = Expand[e6]
```

$\frac{2\,e^{-t}}{3} - 2\,e^{t} + \frac{4\,e^{2t}}{3}$

```
e8 = e7 /. (2 ⅇ^-t/3 - 2 ⅇ^t) → ExpToTrig[2 ⅇ^-t/3 - 2 ⅇ^t]
```

$\frac{4\,e^{2t}}{3} - \frac{4\,\text{Cosh}[t]}{3} - \frac{8\,\text{Sinh}[t]}{3}$

1. Above: The top and bottom green cell expressions match the text answers for y_1 and y_2 respectively.

13. $y_1' = y_2 - 5\,\text{Sin}[t]$
$y_2' = -4\,y_1 + 17\,\text{Cos}[t]$
$y_1[0] = 5,\ y_2[0] = 2$

```
ClearAll["Global`*"]
```

```
e1 = {y1'[t] == y2[t] - 5 Sin[t],
  y2'[t] == -4 y1[t] + 17 Cos[t], y1[0] == 5, y2[0] == 2}
e2 = DSolve[e1, {y1, y2}, t]
```

{y1′[t] == -5 Sin[t] + y2[t], y2′[t] == 17 Cos[t] - 4 y1[t], y1[0] == 5, y2[0] == 2}

$$\left\{\left\{y1 \to \text{Function}\left[\{t\}, \frac{1}{4}(4\,\text{Cos}[2\,t] + 7\,\text{Cos}[t]\,\text{Cos}[2\,t] + 9\,\text{Cos}[2\,t]\,\text{Cos}[3\,t] + 4\,\text{Sin}[2\,t] + 7\,\text{Sin}[t]\,\text{Sin}[2\,t] + 9\,\text{Sin}[2\,t]\,\text{Sin}[3\,t])\right],\ y2 \to \text{Function}\left[\{t\}, \frac{1}{2}(4\,\text{Cos}[2\,t] + 7\,\text{Cos}[2\,t]\,\text{Sin}[t] - 4\,\text{Sin}[2\,t] - 7\,\text{Cos}[t]\,\text{Sin}[2\,t] - 9\,\text{Cos}[3\,t]\,\text{Sin}[2\,t] + 9\,\text{Cos}[2\,t]\,\text{Sin}[3\,t])\right]\right\}\right\}$$

```
e3 = e2[[1, 1, 2, 2]]
```

$$\frac{1}{4}(4\,\text{Cos}[2\,t] + 7\,\text{Cos}[t]\,\text{Cos}[2\,t] + 9\,\text{Cos}[2\,t]\,\text{Cos}[3\,t] + 4\,\text{Sin}[2\,t] + 7\,\text{Sin}[t]\,\text{Sin}[2\,t] + 9\,\text{Sin}[2\,t]\,\text{Sin}[3\,t])$$

```
e4 = Simplify[e3]
```

4 Cos[t] + Cos[2 t] + Sin[2 t]

```
e5 = e2[[1, 2, 2, 2]]
```

$$\frac{1}{2}(4\,\text{Cos}[2\,t] + 7\,\text{Cos}[2\,t]\,\text{Sin}[t] - 4\,\text{Sin}[2\,t] - 7\,\text{Cos}[t]\,\text{Sin}[2\,t] - 9\,\text{Cos}[3\,t]\,\text{Sin}[2\,t] + 9\,\text{Cos}[2\,t]\,\text{Sin}[3\,t])$$

```
e6 = Simplify[e5]
```

2 Cos[2 t] + Sin[t] - 2 Sin[2 t]

1. Above: The top and bottom green cell expressions match the text answers for y_1 and y_2 respectively.

15. $y_1' = y_1 + 2\,y_2 + e^{2\,t} - 2\,t$
$y_2' = -y_2 + 1 + t$
$y_1[0] = 1,\ y_2[0] = -4$

```
ClearAll["Global`*"]
```

```
e1 = {y1'[t] == y1[t] + 2 y2[t] + e^(2 t) - 2 t,
   y2'[t] == -y2[t] + 1 + t, y1[0] == 1, y2[0] == -4}
e2 = DSolve[e1, {y1, y2}, t]
```

$\{y1'[t] == e^{2t} - 2t + y1[t] + 2\,y2[t],\; y2'[t] == 1 + t - y2[t],\; y1[0] == 1,\; y2[0] == -4\}$

$\{\{y1 \to \text{Function}[\{t\}, -\frac{e^{-t}\left(4 - e^{2t} e^{t} - 2 e^{2t} - 8\,\text{Log}[e] + 6 e^{2t}\,\text{Log}[e]\right)}{-1 + 2\,\text{Log}[e]}],\; y2 \to \text{Function}[\{t\}, e^{-t}\left(-4 + e^{t} t\right)]\}\}$

```
e3 = e2[[1, 1, 2, 2]]
```

$-\frac{e^{-t}\left(4 - e^{2t} e^{t} - 2 e^{2t} - 8\,\text{Log}[e] + 6 e^{2t}\,\text{Log}[e]\right)}{-1 + 2\,\text{Log}[e]}$

```
e4 = e3 /. (e^-t (4 - e^(2 t) e^t - 2 e^(2 t) - 8 Log[e] + 6 e^(2 t) Log[e])) →
    Expand[e^-t (4 - e^(2 t) e^t - 2 e^(2 t) - 8 Log[e] + 6 e^(2 t) Log[e])]
```

$-\frac{-e^{2t} + 4 e^{-t} - 2 e^{t} - 8 e^{-t}\,\text{Log}[e] + 6 e^{t}\,\text{Log}[e]}{-1 + 2\,\text{Log}[e]}$

```
e5 = FullSimplify[e4]
```

$\frac{e^{2t} + 2\,\text{Cosh}[t]\,(-1 + \text{Log}[e]) + (6 - 14\,\text{Log}[e])\,\text{Sinh}[t]}{-1 + 2\,\text{Log}[e]}$

```
e6 = e5 /. Log[e] → 1
```

$e^{2t} - 8\,\text{Sinh}[t]$

```
e7 = e6 /. (-8 Sinh[t]) → TrigToExp[-8 Sinh[t]]
```

$e^{2t} + 4 e^{-t} - 4 e^{t}$

```
e8 = e2[[1, 2, 2, 2]]
```

$e^{-t}\left(-4 + e^{t} t\right)$

```
e9 = Expand[e8]
```

$-4 e^{-t} + t$

1. Above: The top and bottom green cell expressions match the text answers for y_1 and y_2 respectively.

17 - 20 Network
Find the currents in the below circuit diagram for the following data, showing the details.

17. $R_1 = 2\,\Omega,\; R_2 = 8\,\Omega,\; L = 1\,H,\; C = 0.5\,F,\; E = 200\,V$

19. In problem 17 find the particular solution when currents and charge at t=0 are zero.

Out[118]=